# NUMERICAL ANALYSIS OF THE EFFECT OF LASER RADIATION ON THE PLASMA OF A WELDING ARC 

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UDC 621.791.75:621.373.826; 621.9.048.7

The process of interaction between laser radiation and the plasma of a welding arc is investigated numerically. The equations of continuity, momentum, and energy of viscous flow and the equations of electric-current flow and laser-radiation transfer are employed.

Gas-shielded consumable-electrode and nonconsumable-electrode arc welding is one of the most inexpensive and efficient methods for producing permanent joints [1]. Having a whole number of advantages it is, however, not devoid of drawbacks such as a slow rate of welding, a large zone of thermal effect, the arc's low stability at small currents, a strong dependence of the parameters of the weld on the arc's length, etc.

Laser welding improves the quality of joints significantly as compared to arc welding. The zone of the thermal effect decreases substantially, the high rate of cooling makes it possible to decrease the grain growth, which ensures the high-strength properties of the compounds. Laser welding's significant drawback is its low efficiency: $8-10 \%$ for steel, and still less for materials with a high surface reflectivity.

In this connection, increasing attention is being paid to combined laser-arc methods of welding, heat treatment, and cutting [2]. Underlying them is the principle of the joint use of the energy of the electric arc and laser radiation, which makes it possible to combine the quality of laser welding joints with the economy of arc processes. Besides, it is possible to control the energy transfer from the laser-arc heating source to the material processed.

To construct a mathematical model that reflects the real process, the interaction between the laser radiation and the electric arc, is considered in the space bounded by the electrode on one side and by the surface of the workpiece on the other. The scheme of the process has an axis of symmetry, on which a hollow cylindrical electrode is positioned. An electric arc is initiated between the electrode and the surface, which is at some distance from it and perpendicular to the axis of symmetry. Through a hole in the electrode and from its external side, a shielding gas is fed that flows onto the wall, spreads along it, and leaves the region under consideration. Simultaneously, the focused radiation beam of a continuous-wave laser propagates along the axis through the hole in the electrode into the region under consideration. Its focus can be set in the space between the electrode and the surface, on the surface, or behind it.

The motion of the gas (plasma) is described in a formulation for laminar flow of a viscous compressible fluid. The continuity, momentum, and energy equations for the current function, vortex stress, and temperature in a cylindrical system of coordinates have the form

$$
\begin{gather*}
\frac{\partial}{\partial z}\left(\frac{1}{\rho r} \frac{\partial \psi}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{\rho r} \frac{\partial \psi}{\partial r}\right)=-\Omega  \tag{1}\\
r^{2}\left(\frac{\partial}{\partial z}\left(\frac{\Omega}{r} \frac{\partial \psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(\frac{\Omega}{r} \frac{\partial \psi}{\partial z}\right)\right)-\frac{\partial}{\partial z}\left(r^{3} \frac{\partial}{\partial z}\left(\mu \frac{\Omega}{r}\right)\right)-\frac{\partial}{\partial r}\left(r^{3} \frac{\partial}{\partial r}\left(\mu \frac{\Omega}{r}\right)\right)- \\
-r^{2}\left(\frac{\partial}{\partial z}\left(\frac{v_{z}^{2}+v_{r}^{2}}{2}\right) \frac{\partial \rho}{\partial r}-\frac{\partial}{\partial r}\left(\frac{v_{z}^{2}+v_{r}^{2}}{2}\right) \frac{\partial \rho}{\partial z}\right)=0 \tag{2}
\end{gather*}
$$

Central Scientific-Research Institute of Structural Materials "Prometei," St. Petersburg, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 5, pp. 951-957, September-October, 1999. Original article submitted August 25, 1998; revision submitted January 26, 1997.

$$
\begin{gather*}
\frac{\partial}{\partial z}\left(c_{p} T \frac{\partial \psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(c_{p} T \frac{\partial \psi}{\partial z}\right)-\frac{\partial}{\partial z}\left(\frac{\mu}{\sigma} r \frac{\partial}{\partial z}\left(c_{p} T\right)\right)-\frac{\partial}{\partial r}\left(\frac{\mu}{\sigma} r \frac{\partial}{\partial r}\left(c_{p} T\right)\right)+ \\
+r\left(\sigma_{j} E^{2}+\mu_{\omega} P-q\right)=0 \tag{3}
\end{gather*}
$$

where

$$
\nu_{z}=\frac{1}{\rho r} \frac{\partial \psi}{\partial r} ; \quad v_{r}=-\frac{1}{\rho r} \frac{\partial \psi}{\partial z} ; \quad \Omega=\left(\frac{\partial v_{r}}{\partial z}-\frac{\partial v_{z}}{\partial r}\right)
$$

The electric processes in laser-arc welding are determined by the law of conservation of electric charges and the law of electromagnetic induction:

$$
\begin{equation*}
\operatorname{div} j=0, \operatorname{rot} E=0, j=\sigma_{j} E \tag{4}
\end{equation*}
$$

The introduction of $I$ as a variable

$$
j_{z}=\frac{1}{r} \frac{\partial I}{\partial r}, \quad j_{r}=-\frac{1}{r} \frac{\partial I}{\partial z}
$$

which is related to the current in the arc by the dependence $I_{\text {arc }}=2 \pi I$, makes it possible to represent the equation for the current in the arc:

$$
\begin{equation*}
\frac{\partial}{\partial z}\left(\frac{1}{\sigma_{j} r} \frac{\partial I}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{\sigma_{j} r} \frac{\partial I}{\partial r}\right)=0 \tag{5}
\end{equation*}
$$

The laser-radiation field propagating in the volume investigated is written in the form of a quasiplane electromagnetic wave, wherein the energy flow is directed along the $z$ axis. In a parabolic approximation, it has the form [3]

$$
\begin{equation*}
2 i k \frac{\partial U}{\partial z}+\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial U}{\partial r}\right)+k^{2}\left(-\frac{n_{\mathrm{e}}}{n_{\mathrm{cr}}}+\frac{i \mu_{\omega}}{k}\right) U=0 \tag{6}
\end{equation*}
$$

In this equation, the first term in parentheses reflects the refraction of the laser beam in the plasma of the electric arc, while the second term reflects absorption. The plasma is considered to be in equilibrium and the concentration of free electrons for a given temperature is found from the Saha equation.

The laser-beam intensity is the time-averaged value of the axial component of the density of the flow of the beam's electromagnetic energy and is related to the complex amplitude of the electric field strength of the laser radiation by [4]

$$
\begin{equation*}
P=\frac{1}{2}\left(\frac{\varepsilon^{0}}{\mu^{0}}\right)^{0.5} U^{2} \tag{7}
\end{equation*}
$$

The plasma-radiation losses in a continuous spectrum with allowance for recombination to the ground level are given by the expression $[5,6]$

$$
\begin{equation*}
q=\frac{280 p^{2} \alpha^{2}}{\left(T / 10^{4}\right)^{5 / 2}}\left(1+0.027 \frac{T}{10^{4}}\right) \tag{8}
\end{equation*}
$$

Thus, system of Eqs. (1)-(3) and (5)-(8) describes the processes in the interelectrode gap in the laser-arc discharge. It is supplemented by the equations of gas state and the transport coefficients and the optical properties of the plasma as functions of temperature and pressure. The boundary conditions for a given system of equations are chosen in accordance with the formulated physical model.

On the axis of symmetry $r=0$ :

$$
\psi=0 ; \Omega=0 ; \frac{\partial T}{\partial r}=0 ; \frac{\partial I}{\partial r}=0 ; \frac{\partial U}{\partial r}=0
$$

At the external boundary of the calculated region $r=R$ :

$$
\frac{\partial \psi}{\partial r}=0 ; \frac{\partial \Omega}{\partial r}=0 ; \frac{\partial T}{\partial r}=0 ; \frac{\partial I}{\partial r}=0 ; \frac{\partial U}{\partial r}=0
$$

At the right boundary of the calculated region $z=L$, where the workpiece is placed:

$$
\psi=0 ; \Omega=\Omega_{\mathrm{w}} ; \quad \frac{\partial T}{\partial z}=0 ; \frac{\partial I}{\partial z}=0 ; \quad \frac{\partial U}{\partial z}=0
$$

Here and in what follows $\Omega_{\mathrm{w}}$ is used to refer to special conditions for a vortex on the wall [8]. On a hollow electrode of radius $R_{2}$ and length $L_{1}$ with an axial opening of radius $R_{1}$ :
when $r=R_{1}$ and $r=R_{2}$ for $0<z<L_{1}$

$$
\psi=0.5 \rho_{0} V_{1} R_{1}^{2} ; \quad \Omega=\Omega_{\mathrm{w}} ; \frac{\partial T}{\partial z}=\mathrm{const} ; I=\mathrm{const} ; \quad \frac{\partial U}{\partial z}=0
$$

when $z=L_{1}$ for $R_{1}<r<R_{2}$

$$
\psi=0.5 \rho_{0} V_{1} R_{1}^{2} ; \quad \Omega=\Omega_{w} ; \quad \frac{\partial T}{\partial z}=0 ; I=0.5 j_{0}\left(r^{2}-R_{1}^{2}\right) ; \quad \frac{\partial U}{\partial z}=0
$$

The shielding gas is fed through two coaxial openings at the left boundary, the laser radiation entering through one of them along the axis of symmetry. The boundary conditions for them have the form:
when $z=0$ for $0<r<R_{1}$

$$
\psi=0.5 \rho_{0} V_{1} r^{2} ; \quad \Omega=0 ; \quad T=T_{0} ; \quad I=0 ; \quad U=U\left(r_{\mathrm{F}}, r\right)
$$

when $z=0$ for $R_{2}<r<R_{3}$

$$
\psi=0.5 \rho_{0}\left(V_{1} R_{1}^{2}+V_{2}\left(r^{2}-R_{2}^{2}\right)\right) ; \quad \Omega=0 ; \quad T=T_{0} ; \quad I=0.5 j_{0}\left(R_{2}^{2}-R_{1}^{2}\right) ; \quad U=0
$$

when $r=R_{3}$ for $0<z<L_{2}$

$$
\psi=0.5 \rho_{0}\left(V_{1} R_{1}^{2}+V_{2}\left(R_{3}^{2}-R_{2}^{2}\right)\right) ; \Omega=\Omega_{w} ; \frac{\partial T}{\partial r}=0 ; \quad I=0.5 j_{0}\left(R_{2}^{2}-R_{1}^{2}\right) ; \quad U=0
$$

when $z=L_{2}$ for $R_{3}<r<R$

$$
\psi=0.5 \rho_{0}\left(V_{1} R_{1}^{2}+V_{2}\left(R_{3}^{2}-R_{2}^{2}\right)\right) ; \quad \Omega=\Omega_{\mathrm{w}} ; \frac{\partial T}{\partial z}=0 ; \quad I=0.5 j_{0}\left(R_{2}^{2}-R_{1}^{2}\right) ; \quad U=0
$$

To prescribe the initial distribution of the complex amplitude of the laser-radiation field, it is assumed that a Gaussian laser-radiation beam focused by an optical system is introduced through the opening in the electrode, the beam in the absence of plasma having the minimum width $r_{F}$ in the focal plane $z=F$. The spatial distribution of the complex amplitude of the field of this beam is determined by the expression [7]

$$
U\left(r_{\mathrm{F}}, r\right)=U_{\mathrm{F}} \frac{r_{\mathrm{F}}}{r_{z}} \exp \left[-\frac{r^{2}}{r_{z}^{2}}+i\left(\frac{k r^{2}}{2 R_{z}}-\varphi_{z}\right)\right]
$$

where

$$
r_{z}^{2}=r_{\mathrm{F}}^{2}\left[1+(z-F)^{2} / z_{\mathrm{F}}^{2}\right] ; R_{z}=(z-F)\left[1+z_{\mathrm{F}}^{2} /(z-F)^{2}\right]
$$




Fig. 1. Field of temperatures $T$ in arc column for current $I_{\text {arc }}=100 \mathrm{~A}$ (a) $Q$ $=0$; b) 1.5 kW : 1) $T=350 \mathrm{~K}$; 2) 500 ; 3) 1000 ; 4) 2000 ; 5) 3000 ; 6) 5000 ; 7) 7000 ; 8) 8000 ; 9) 9000 ; 10) 10,000 ; 11) 12,000 ; 12) 15,000 ; 13) 18,000 ; 14) 20,000 .

$$
\varphi_{z}=\arctan \left[(z-F) / z_{\mathrm{F}}\right] ; z_{\mathrm{F}}=k r_{\mathrm{F}}^{2} / 2
$$

The constant $U_{\mathrm{F}}$ is found from the integral relation for the total power of radiation $Q$ :

$$
U_{\mathrm{F}}=\left[4 Q\left(\mu^{0} / \varepsilon^{0}\right)^{0.5} /\left(\pi r_{\mathrm{F}}^{2}\right)\right]^{0.5}
$$

System of Eqs. (1)-(3) and (5)-(8) together with the given boundary conditions fully determine the temperature, the plasma velocity, the electrical parameters of the arc that is exposed to laser radiation, and the spatial distribution of the intensity of the laser beam that is interacting with it. This system of equations was solved numerically by the finite-difference method of a reference volume [8]. The difference analog of Eqs. (1)-(3) and (5) is constructed according to the counterflow scheme [8] while (6) is approximated by central differences [3]. The system of algebraic equations obtained was calculated by the Gauss-Seidel iteration method. The numerical procedure involved a separate solution of the gas-dynamic and electric-current equations (1)-(3) and (5) and the equation for the amplitude of the laser-beam field (6). Initially, we calculated Eqs. (1)-(3) and (5) until the required convergence of $10^{-3}$ was attained and then we solved Eq. (6) alternately up to the prescribed accuracy, following which in 100 iterations we found again the unknown quantities of Eqs. (1)-(3), (5), etc., now until the prescribed convergence in all the variables was attained. Taking into account the smallness of the cross section of the laser beam as compared to the radius of the region under consideration, we took the radius of the internal channel of the electrode $R_{1}$ as the upper limit of integration for Eq. (6), the radius of the incoming beam also being smaller than this dimension.

A numerical analysis was carried out for a case that modeled laser-arc argon welding. In the calculation, the arc current varied in the range $50 \mathrm{~A}<I_{\text {arc }}<200 \mathrm{~A}$; the power of the laser beam with wavelength $\lambda=10.5 \mu \mathrm{~m}$ changed to $Q=1.5 \mathrm{~kW}$. The minimum radius of the beam $r_{\mathrm{F}}=0.0002 \mathrm{~m}$ was located on the workpiece surface while the angle of focusing was taken to be $5^{\circ}$ or $10^{\circ}$. The electrode has inside radius $R_{1}=0.002 \mathrm{~m}$, outside radius $R_{2}$ $=0.003 \mathrm{~m}$, and the electrode path into the calculated region $L_{1}=0.002 \mathrm{~m}$. The radius of the external nozzle is $R_{3}$ $=0.008 \mathrm{~m}$. The calculated region has length $L=0.007 \mathrm{~m}$ and radius $R=0.012 \mathrm{~m}$. The inlet velocity of the gas (argon) was taken to be $V_{1}=0.2 \mathrm{~m} / \mathrm{sec}$ through the electrode and $V_{2}=1 \mathrm{~m} / \mathrm{sec}$ through the external nozzle. The inlet temperature of the gas is $T_{0}=300 \mathrm{~K}$. The pressure is equal to atmospheric pressure. A relative accuracy of $10^{-5}$ for the complex amplitude of the laser-beam field and of no worse than than $10^{-3}$ for the remaining variables was attained in all the computations.

The results of the numerical investigation of the characteristics of the plasma of an arc-discharge column exposed to laser radiation showed that the main physical mechanism of this process is additional heating of the plasma by the laser beam (Fig. 1). The temperature of the central region of the plasma increases substantially as


Fig. 2. Field of electric-current density $j\left(\mathrm{~A} / \mathrm{m}^{2}\right)$ in arc column for current $I_{\text {arc }}$ $=100 \mathrm{~A}$ (a) $Q=0$; b) 1 kW ):1) $j=10^{4}$; 2) $10^{5}$; 3) $5 \cdot 10^{5}$; 4) $10^{6}$; 5) $3 \cdot 10^{6}$; 6) $5 \cdot 10^{6}$; 7) $7 \cdot 10^{6}$; 8) $10^{7}$; 8) $3.5 \cdot 10^{7}$; 10) $4.5 \cdot 10^{7}$.


Fig. 3. Variation in radial profiles of radiation intensity $P$ along length of interelectrode gap for $Q=0.5 \mathrm{~kW}$ ( $I_{\mathrm{arc}}=200 \mathrm{~A}$, solid curves; $I_{\text {arc }}=0$, dashed curves).
a result of its local heating due to absorption of the laser-radiation energy. The plasma temperature increases from $T=10,000-12,000 \mathrm{~K}$, which is characteristic of a conventional arc, to $T=18,000-21,000 \mathrm{~K}$ under the action of radiation, the maximum temperature increasing with the laser-beam power. The temperature increase increases the degree of ionization and electrical conductivity of the plasma, which leads to a redistribution of the current in it (Fig. 2). Depending on the power of the laser radiation, the density of the current in the control part of the arc increases by a factor of $7-8$ while the voltage drop on the arc column decreases by a factor of 1.5-2.0. This spatial distribution of $j$ causes up to $80 \%$ of the entire current to flow through the central part of the arc, although in practice its external boundary remains in place. The characteristics of the plasma in laser-arc interaction depend not only on the laser-radiation power but also on the flowing current. Its increase, all other things being equal, causes the beginning of the laser-arc interaction to approach the inlet section. A variation in the thermal regime of the arc due to the action of laser radiation causes no substantial redistribution of gas-dynamic characteristics of the plasma. Thus, for $I_{\mathrm{arc}}=100 \mathrm{~A}$ and $Q=1.5 \mathrm{~kW}$, the increase in axial and radial velocities of the flow does not exceed $3-5 \%$ as compared to a conventional arc with the same current. The refractive index of the argon plasma and the absorption coefficient of the laser radiation depend substantially on temperature [9]; therefore, because of the high nonuniformity of the temperature of the plasma in interaction between the electric arc and the laser radiation, its optical properties will also be spatially nonuniform. When the laser beam passes through this optically nonuniform absorbing medium, the distribution of the radiation intensity changes substantially due to refraction and attenuation of the beam in the plasma (Fig. 3), and the arc-current strength and the laser-radiation power have a substantial effect on these parameters. For example, an increase in the current causes the energy absorption and the refraction broadening of the beam that passes through the plasma to begin earlier along $z$; an increase in


Fig. 4. Distribution of laser-radiation intensity $P\left(\mathrm{~W} / \mathrm{m}^{2}\right)$ along beam axis with power $Q=1.5 \mathrm{~kW}$ and angle of focusing of $5^{\circ}$ for different arc currents: 1) $\left.\left.I_{\text {arc }}=0 ; 2\right) 100 ; 3\right) 200 \mathrm{~A}$.
the beam power has a similar action. A variation in the angle of focusing also has an effect on the distribution of the laser-radiation intensity along the length of the interelectrode gap. For instance, a decrease in the angle of focusing of from 10 to $5^{\circ}$ in certain combinations of laser-beam power and arc current brings about the following situation: the initial decrease in the radiation intensity due to absorption and refraction gives way to its increase (Fig. 4) and, starting with a coordinate $z, P$ exceeds its value in an undisturbed beam and self-focusing of the beam occurs. This results from the complicated dependence of the complex dielectric constant of the argon plasma on temperature [ 9 ]. Below $T \sim 16,500 \mathrm{~K}$, the minimum of the dielectric constant and hence of the refractive index is on the axis of the beam and the electric arc, which causes defocusing of the beam, since the peripheral layers of the plasma have a large refractive index. A temperature increase over $16,500 \mathrm{~K}$ on the axis due to the absorption of radiation causes the minimum of the dielectric constant to shift from the axis to the periphery and near the beam axis a region is formed that acquires the role of a collecting lens for the neutral part of the beam. As the temperature increases, this zone expands, covering an increasing part of the cross section of the laser beam, and the beam begins to be focused over the entire cross section. The effective radius of the beam decreases to the extent that, in spite of the absorption of the beam in the plasma, the radiation intensity on the axis exceeds its value in the undisturbed laser beam.

The results of numerical modeling of the interaction between laser radiation and an argon welding arc showed that the space characteristics of the plasma differ significantly from its parameters in the absence of a laser beam. A laser beam passing through the plasma is partially absorbed and, under certain conditions, can be focused additionally. Therefore, using focused laser radiation we can control quite effectively the characteristics of the electric-arc column and, conversely, adjust the focusing of the laser beam by changing the arc current. An analysis of the results obtained indicates a number of practical advantages of the realization of this process: the increase in plasma conductivity along the axis must lead to stabilization of the conducting region relative to the beam axis and improvement of the stability of the characteristics of the arc's plasma column and hence to extension of the range of stable operating regimes; the decrease in the voltage drop on the laser-arc discharge reduces the hazard of double arc formation, which will make it possible to use longer channels, and the joint action of the electric arc and the self-focusing laser radiation on the treated material must increase the penetration depth.

## NOTATION

$z$ and $r$, coordinates, $\mathrm{m} ; \psi$, current function, $\mathrm{kg} / \mathrm{sec} ; \Omega$, vortex, $1 / \mathrm{sec} ; T$, temperature, $\mathrm{K} ; E$, electric-field strength, $\mathrm{V} / \mathrm{m} ; P$, radiation intensity, $\mathrm{W} / \mathrm{m}^{2} ; q$, radiation heat losses, $\mathrm{W} / \mathrm{m}^{3} ; \rho$, density, $\mathrm{kg} / \mathrm{m}^{3} ; \mu$, viscosity, $\mathrm{N} \cdot \mathrm{sec} / \mathrm{m}^{2} ; v$, velocity, $\mathrm{m} / \mathrm{sec} ; \sigma$, Prandtl number; $c_{p}$, specific heat, $\mathrm{J} /(\mathrm{kg} \cdot \mathrm{K}) ; \sigma_{j}$, specific electrical conductivity, $1 /(\Omega \cdot \mathrm{m}) ; \mu_{\omega}$, absorption coefficient, $1 / \mathrm{m} ; j$, current density, $\mathrm{A} / \mathrm{m}^{2} ; I$, electric-current function, $\mathrm{A} ; I_{\text {arc }}$, arc current, $\mathrm{A} ; k$, wave number; $U$, complex amplitude of electric field; $n_{e}$, electron concentration, $1 / \mathrm{m}^{2} ; n_{\mathrm{cr}}$, critical electron concentration, $1 / \mathrm{m}^{3} ; i$, imaginary unit; $\varepsilon^{0}$, electric constant, $\mathrm{f} / \mathrm{m} ; \mu^{0}$, magnetic constant, $\mathrm{H} / \mathrm{m} ; p$, pressure, $\mathrm{N} / \mathrm{m}^{2}$; $\alpha$, degree of ionization; $Q$, laser-radiation power, $\mathrm{W} ; V_{1}$ and $V_{2}$, inlet velocity of gas, $\mathrm{m} / \mathrm{sec} ; R_{1}, R_{2}$, and $R_{3}$, radial
dimensions of electrode and external nozzle, $m ; L_{1}$, electrode path into calculated region, $m ; L_{2}$, path of external nozzle into calculated region, $\mathrm{m} ; L$ and $R$, geometric dimensions of calculated region, $\mathrm{m} ; F$, distance from $z=0$ to the focus. Subscripts and superscrips: 0,1 , and 2 , conditions at the inlet; $w$, on the wall; $F$, focal plane; $r$ and $z$, coordinate directions.

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